Validating Quantum Electrodynamics' Interaction of Radiation with Matter

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Quantization of the electromagnetic field laws of classical electrodynamics appear in the form of

$$F_{\mu v_l p} + F_{vp\mu} + F_{p\mu_l v} = 0 \qquad \text{Eq. (1a)}$$

$$F_{v_l v=0} \qquad \text{Eq. (1b)}$$

where $F_{\mu v}$ are components of field, E and H. There is a (Lagrangian) derivation

$$L = {}^1\!/_4 \; F_{\mu v} \, F_{\mu v} \qquad \text{Eq. (2)}$$

It's more clearly understood to work with potentials A_μ defined by

$$F_{\mu v} = A_{v,\mu} - A_{\mu v} \qquad \text{Eq. (3)}$$

<u>Note:</u> The Greek subscript symbols run 1 to 4 for vectors or operators taking the value 0, 1, 2, 3, and the Latin indices run from 1 to 3 in the coordinates.

Also

$$K = (K,\ K_0) + (K_1,\ K_2,\ K_3,\ K_0),$$

$$X = (X,\ X_0) + (X_1,\ X_2,\ X_3,\ X_0).$$

The same holds for $p, q, K_6 = \omega(omega)$, = it and the units are $h = c = 1$ which satisfies the equation of motion

$$\Box A_\mu(x) - A_v, \mu_v(x) = 0 \qquad \text{Eq. (4)}$$

From Equation (3) A is determined up to a gradient of a scalar function $\Lambda(X)$ i.e. $A_{\mu}{}^{l} = A_{\mu} + \Lambda_{l}\mu$ (gauge transformation). The fields $F_{\mu v}$ and Equation (4) remain invariant.

Simplifying Equation (4) by introducing a (Lorentz) condition on the potentials,

$$A_{\mu l \mu} = 0 \qquad \text{Eq. (6)}$$

reduces Equation (4) to

$$\Box A_{\mu} = 0 \qquad \text{Eq. (7)}$$

which together with Equation (6) is equivalent to Maxwell's equations.

<u>However</u>, condition (7) still does not determine uniquely A_{μ}, but now the gauge transformation restricts the class of function Λ to solutions of the equation

$$\Box \Lambda = 0 \qquad \text{Eq. (8)}$$

on account of photon mass = 0 the fields $F_{\mu v}$ and not the potentials A_{μ} have a direct physical meaning this however is not the case if $m \neq 0$.

The canonical (momenta) variables $\pi_{\mu}(X)$ corresponding to Equation (3)

are

$$\pi_{\mu} = 1\, F_{4\mu}\,(X) \qquad \text{Eq. (9)}$$

Thus $\pi_4 = 0$ so Equation (9) cannot be solved by $A_{\mu\alpha}$. Now it is evident Equation (2) is invalid. How do we then validate the quantization of free (so-called Maxwellian) fields?

Solution:

The Lagrangian Equation (3) i s replaced by

$$L = -{}^{1}/_{2}\; F_{\mu v}F_{\mu v} - {}^{1}/_{2}\, A_{\mu l} m A_{v l v} = -{}^{1}/_{2}\, A_{\mu l v}A_{v l \mu} \qquad \text{Eq . (10)}$$

from which the canonical momenta are given by

$$\pi_l(X) = 1\ F_4(X) \qquad \text{Eq. (11a)}$$

and

$$\pi_4(X) = lA_{v,v}(X) \qquad \text{Eq. (11 b)}$$

and hence instead of Equation (8), we obtain the weaker condition

$$\Box A_{v\ v}(X) = 0 \qquad \text{Eq. (12)}$$

which follows from Equation (4).

The invalidity occurs when one takes the general solution of the equation only those which satisfy the initial conditions when $t = 0$.

$A_{v,v}(X) = 0, A_{v,ov}(X) = 0$ for all values of X then from Equation (12)

it follows that $A_{vlv}\ (X)$ vanishes for all t. This is in agreement with the formulation of the theory of electromagnetism.

The above is stated to avoid the expressions describing a conceptual fallacy leading to an enormous number of invalid consequent mathematical expressions describing abstract phenomena in terms of non-abstract imagination, leading to a horrendous misunderstanding of the most profound subject of electrical engineering.